Analyzing Physics-Informed Neural Networks for Solving Classical Flow Problems

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Physics-Informed Neural Networks for Computational Fluid Dynamics (CFD)

- Predictions by purely data-driven Deep Neural Networks (DNNs) can suffer from physical inconsistencies
- Physics-Informed Neural Networks (PINNs) integrate physical laws, governing equations, initial and/or boundary conditions into the loss function to improve the predictive capability of DNNs[1]
- Though PINNs can be trained on limited spatial or temporal data to attain accurate results, data-free PINNs are difficult to train[2]
- Based on the complexity of the problem and the desired accuracy of the solution, hybrid models combining CFD solvers and PINNs have been developed[3]

Problem description and governing equations

The current study juxtaposes PINN-generated flow fields to analytical solutions and compares the predictive capability of PINNs with that of DNNs, which do not have a physical loss as a constraint. The following cases are considered in the current study,

- **Poiseuille flow**[4]

Pressure drop along pipe length

$$\frac{\partial p}{\partial x} = \mu\left(\frac{\partial^{2u}}{\partial r^2} + \frac{1}{r}\frac{\partial u}{\partial r}\right), \frac{\partial p}{\partial r} = 0, -\frac{dp}{dx} = \frac{\Delta p}{L_p} = Gp\,, \mathrm{u(r)} = \frac{G_p}{4\mu}(R_p^2 - r^2)$$

- **Potential flow**

$\nabla \times \vec{u} = 0, \nabla \cdot \vec{u} = 0$, where flow velocity, $\vec{u} = \nabla\emptyset$ ← *Potential function*

Cylinder

$$\emptyset = Ux + \frac{Q}{\pi}\cdot\frac{x}{x^2+y^2}$$

Rankine oval

$$\emptyset = Ux + \frac{m}{4\pi}\cdot log\left[\frac{(x+a)^2+y^2}{(x-a)^2+y^2}\right]$$

- **Blasius boundary layer flow**[5]

$$\frac{\partial \rho u}{\partial x} + \frac{\partial \rho v}{\partial y} = 0$$

$$\rho u\frac{\partial u}{ux} + \rho v\frac{\partial u}{\partial y} = \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right)$$

$$\frac{\partial p}{\partial x} = 0, \frac{\partial p}{\partial y} = 0$$

Scaled stream function and the modified ODE

$$\eta \sim \frac{y}{\delta(x)} = \frac{y}{(\nu x/U_0)^{1/2}}, f(\eta) = \frac{\psi}{(\nu x U_0)^{1/2}}$$

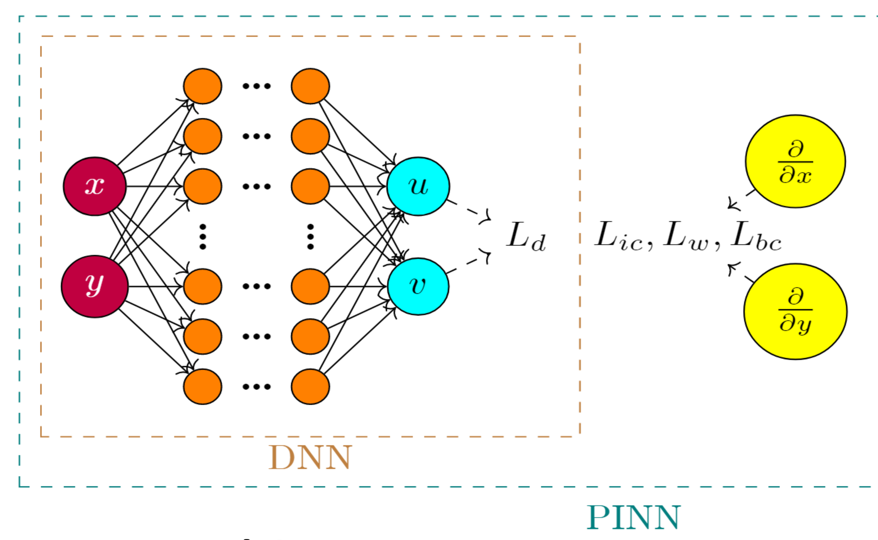
$$\frac{d^3 f}{d\eta^3} + \frac{1}{z} f \frac{d^2 f}{d\eta^2} = 0, f(\eta = 0) = 0,$$

$$f'(\eta = 0) = 0, f'(\eta \to \infty) = 1$$

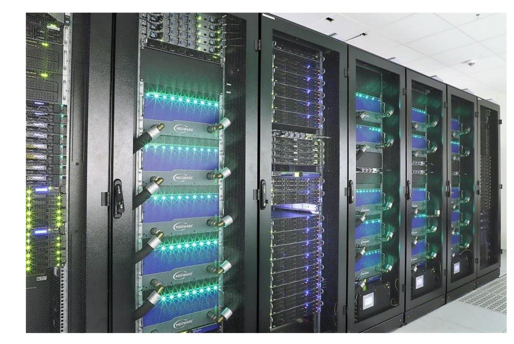
PINNs are computationally expensive compared to DNNs and the achieved accuracy must justify additional computation costs.

Workflow

- **Network architecture for Potential flow**



- **HPC systems**



DEEP-EST-ESB

JURECA-DC

- **Input and Output parameters**

Flow case	Input	Output
Poiseuille flow	r	u
Potential flow	x,y	u,v
Blasius equation	η	f', f''

- **Loss function**

$$L = L_d + L_g$$

$$L_d = \frac{1}{N_d}\sum_{n=1}^{N_d}|\theta - \theta^*|^2, L_g = \frac{1}{N_g}\sum_{n=1}^{N_g}|g(\theta)|^2, \theta \to u, v, f', f''$$

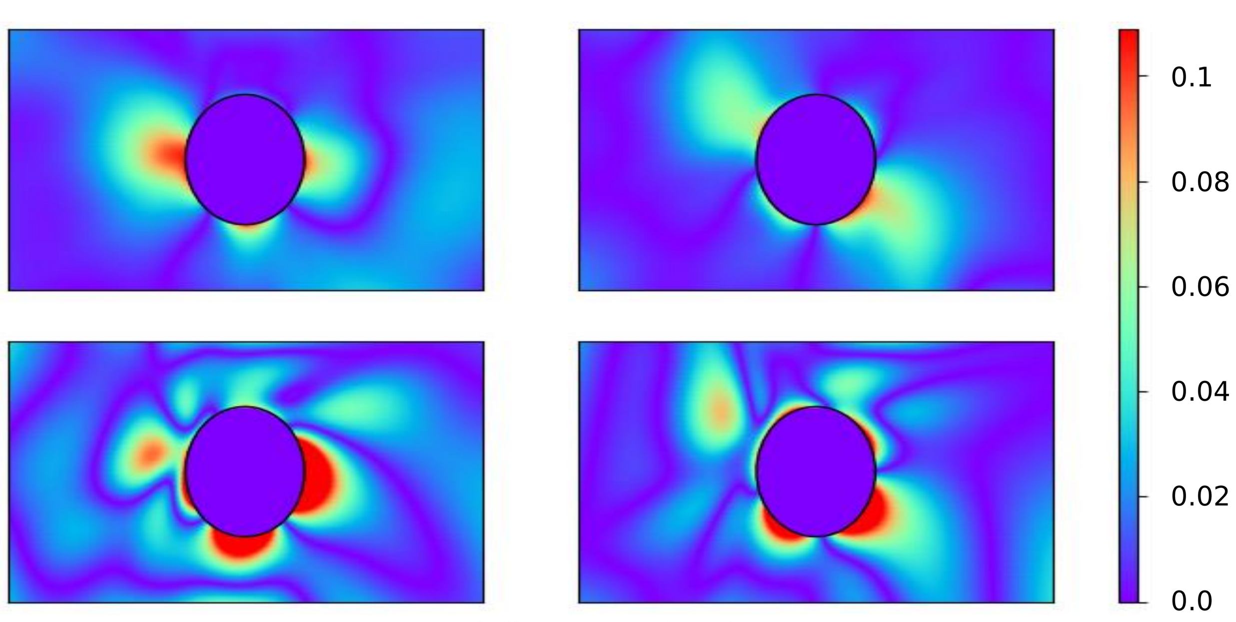
MSE loss from prediction

Loss from governing equations on ic, w, bc data

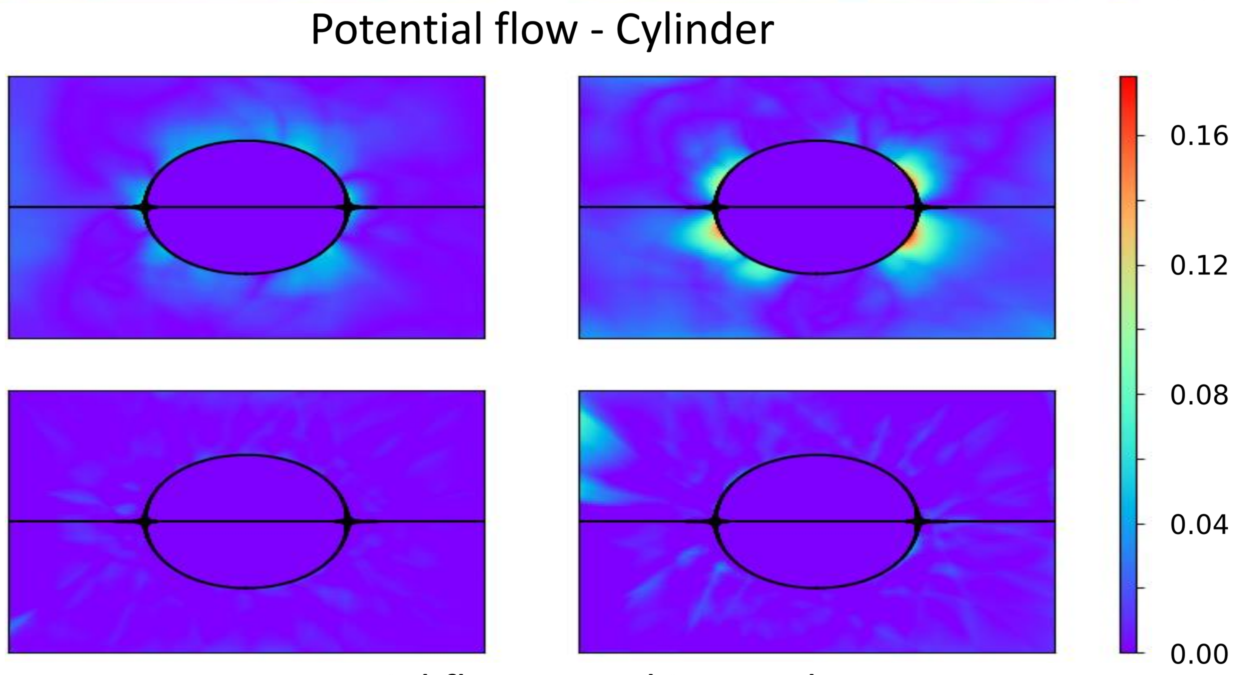
ic, initial conditions
w, wall or body boundary condition
bc, boundary conditions

Results

- **Prediction Error density**[#]



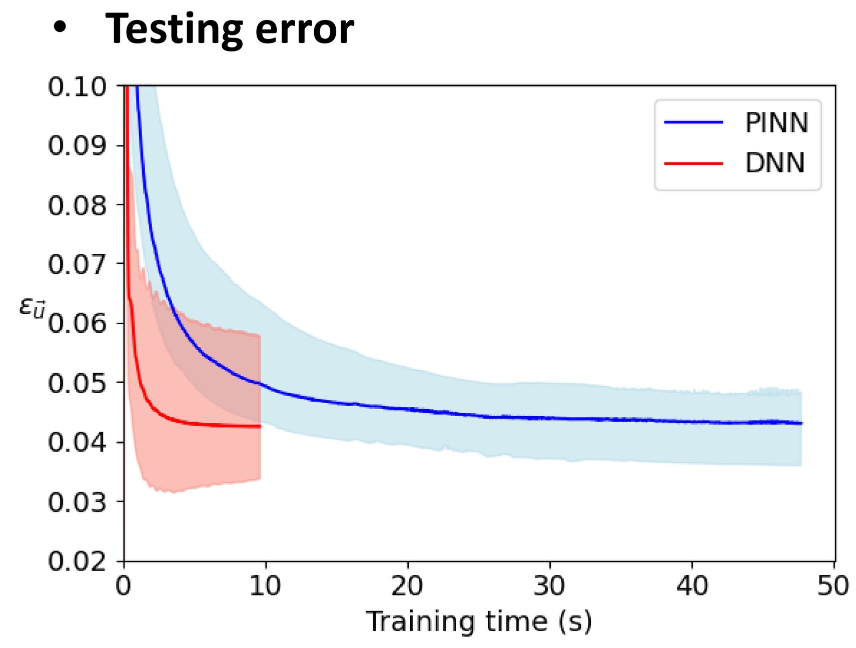
Potential flow - Cylinder



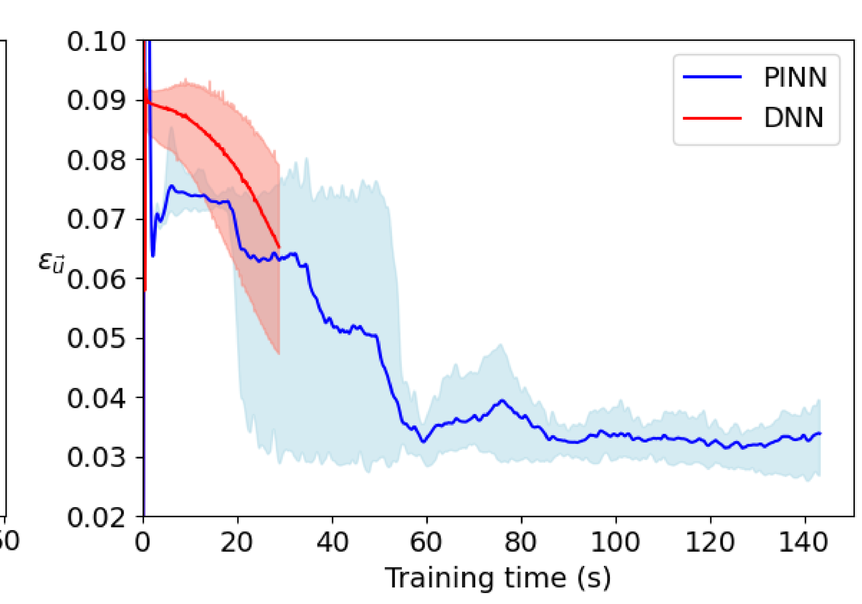
Potential flow – Rankine oval

[#] Columns: Velocity fields u (left) and v (right); Rows: PINN (top) and DNN (bottom)

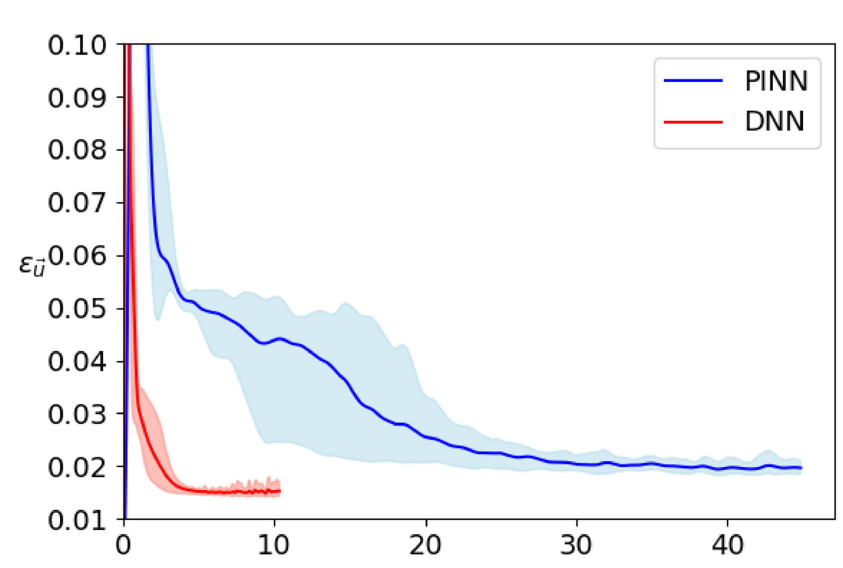
- **Testing error**



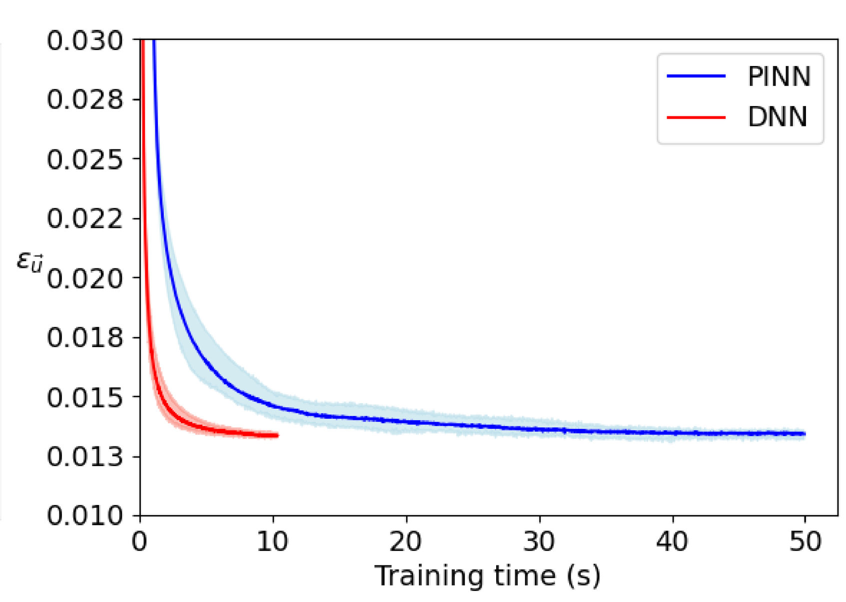
Poiseuille flow



Potential flow Cylinder



Potential flow Rankine oval



Blasius boundary layer flow

Conclusion

- The inclusion of physical constraints in NNs improves the prediction capability of the network implemented for the cases of Poiseuille flow, potential flow around a cylinder, and Blasius boundary layer flow, especially in the near wall flow field
- For the case of potential flow around the Rankine oval, the normalization of the flow data is affected by extreme gradients near the source and sink and the PINN struggles to predict the flow field accurately

Future work

- The prediction capability of PINNs will be evaluated for further types of governing equations, i.e., a lattice-Boltzmann method, and for more complex flow problems, i.e., 2D Taylor-Green vortex
- The effect of constant parameters like the pressure gradient in one-dimensional flow problems will be studied to improve the calculation of the physical loss

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