Analyzing Physics-Informed Neural Networks for Solving Classical Flow Problems

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Problem description and governing equations

The current study juxtaposes PINN-generated flow fields to analytical solutions and compares the predictive capability of PINNs with that of DNNs, which do not obtain a physical loss as a constraint. The following cases are considered in the current study.

- **Poiseuille flow**: Pressure drop along pipe length

\[ \frac{dP}{dz} = \mu \left( \frac{du}{dy} \right) \]

- **Potential flow**: Where flow velocity, \( \mathbf{u} = 0 \)

\[ \nabla \times \mathbf{u} = 0, \quad \nabla \cdot \mathbf{u} = 0 \]

Function potential

\[ \Phi = Ux + \frac{1}{2} \left( x^2 + y^2 \right) \]

- **Blasius boundary layer flow**:

\[ \begin{align*}
\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} &= 0 \\
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} &= 0 \\
\frac{\partial u}{\partial y} &= 0
\end{align*} \]

PINNs are computationally expensive compared to DNNs and the achieved accuracy must justify additional computation costs.

Flow of Physical Solutions to the governing equation

\[ f(x) = \frac{1}{2} \pi \left( 1 - e^{-x^2} \right) \]

- **Prediction Error density**

Results

• Testing error

![Testing error graph](image)

Future work

- The prediction capability of PINNs will be evaluated for further types of governing equations, i.e., a lattice-Boltzmann method, and for more complex flow problems, i.e., 2D Taylor-Green vortex

Conclusion

• The inclusion of physical constraints in NNs improves the prediction capability of the network implemented for the cases of Poiseuille flow, potential flow around a cylinder, and Blasius boundary layer flow, especially in the near wall flow field

• For the case of potential flow around the Rankine oval, the normalization of the flow data is affected by extreme gradients near the source and sink and the PINN struggles to predict the flow field accurately

References


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