NUMERICAL SIMULATION OF GRADUAL COMPACTION OF GRANULAR MATERIALS AND THE UNCERTAINTY QUANTIFICATION OF THE PROPOSED MATHEMATICAL MODEL

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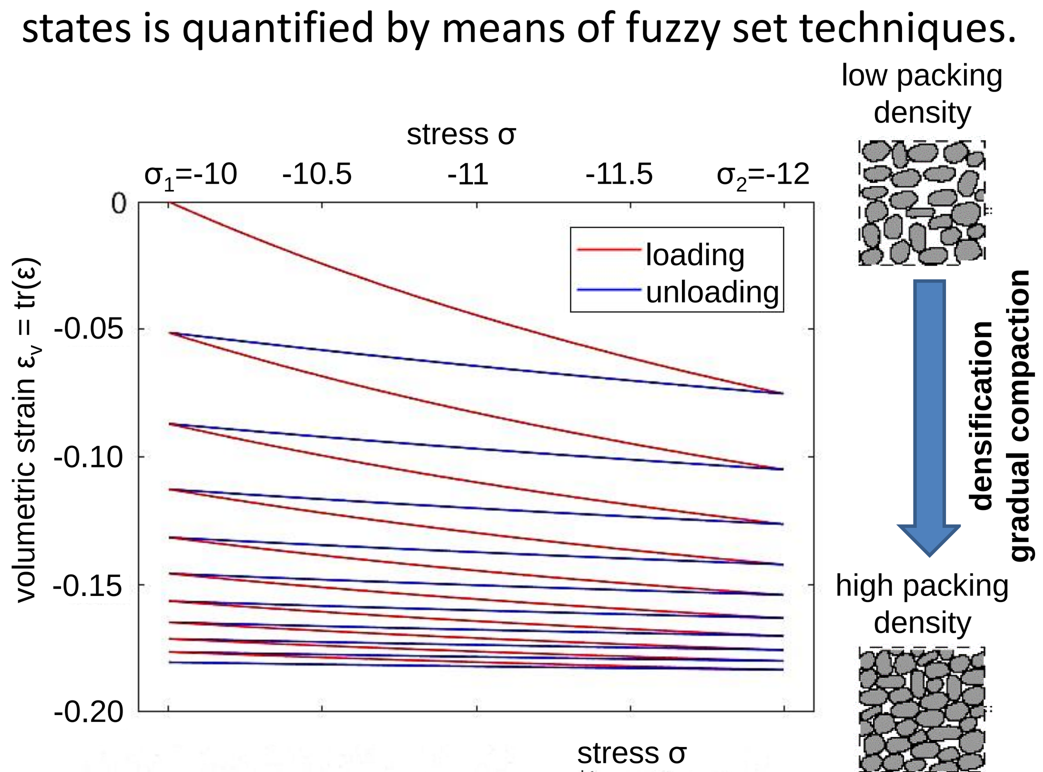
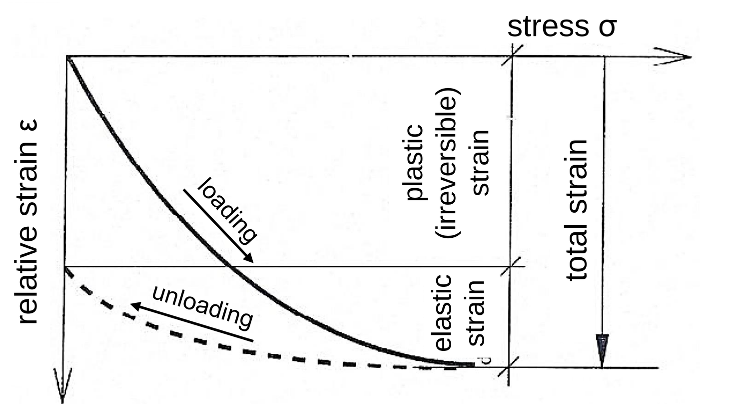
ABSTRACT

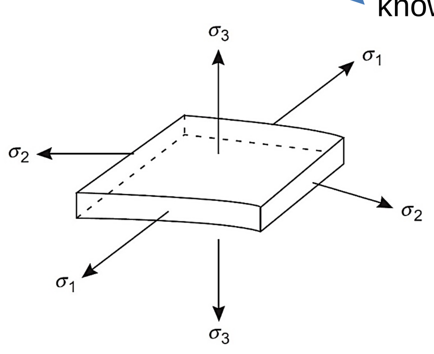
The poster deals with mathematical modelling of granular materials and focuses on the process of their gradual compaction called ratchetting. The model of hypoplasticity introduced by E. Bauer et al. is investigated and the problem of stress-controlled hypoplasticity is considered. The behaviour of strain paths produced by periodic stress paths in a granular material during cyclic loading and unloading is calculated and a limit state of the material is numerically approximated. Then, the impact of uncertain input parameters on ratchetting trends and the material limit states is quantified by means of fuzzy set techniques.





stress-controlled problem

- unknown strain tensor $\boldsymbol{\varepsilon}(t)$
- known stress tensor $\boldsymbol{\sigma}(t) = \sigma(t)\mathbf{S} = \sigma(t)\begin{pmatrix} s_{11} & s_{12} & s_{13} \\ s_{21} & s_{22} & s_{23} \\ s_{31} & s_{32} & s_{33} \end{pmatrix}$

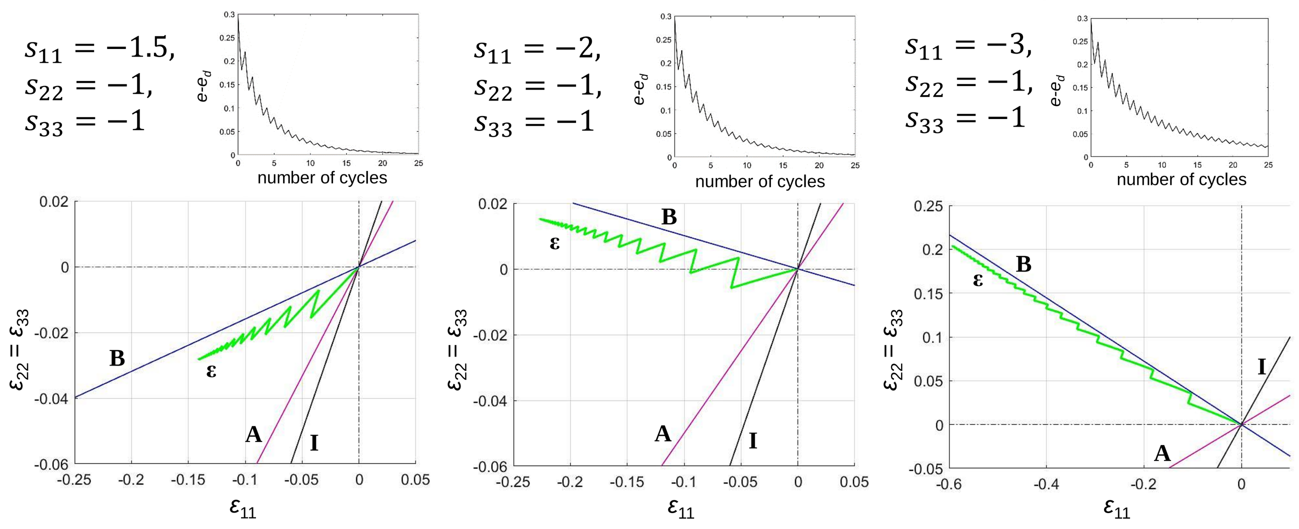


$\sigma(t)$ is a given continuous and alternately increasing (loading period) and decreasing (unloading period) positive $2T$ periodic function between fixed values σ_1 and σ_2, where $0 < \sigma_1 < \sigma_2$

$\sigma(t)$ is defined as $\sigma(t) = \sigma_1 e^{t-t_{2j}}$ for $t \in (t_{2j}, t_{2j+1})$,
$\sigma(t) = \sigma_2 e^{t_{2j+1}-t}$ for $t \in (t_{2j+1}, t_{2j+2})$,
where $T = \log(\sigma_2) - \log(\sigma_1)$, $t_k = kT$ for $k = 0,1,2,\ldots$

$\mathbf{S}$ is a chosen fixed symmetric matrix, $\mathrm{tr}(\mathbf{S}) < 0$, all $s_{ij} \le 0$

ratchetting of $\varepsilon(t)$ under the anisotropic case of loading



MODEL OF HYPOPLASTICITY

- The stress tensor $\boldsymbol{\sigma}$ with a given evolution of $\sigma(t)$ in an interval $t \in [b, b+T]$ is considered, the stress-controlled proportional loading is put in the form $\boldsymbol{\sigma}(t) = \sigma(t)\mathbf{S}$, where $\sigma(t)\colon [b, b+T] \to (0,\infty)$ is a given function and $\mathbf{S}$ is a chosen fixed symmetric tensor,
  - proportional loading if σ is increasing, i.e. $\dot{\sigma} > 0$,
  - proportional unloading if σ is decreasing, i.e. $\dot{\sigma} < 0$.
- An extension of the model from (Bauer et al., 2020 and 2023) is used, where the relation between the stress tensor $\boldsymbol{\sigma}(t)$ and the strain tensor $\boldsymbol{\varepsilon}(t)$ is governed by the rate independent equation, which is rewritten after manipulations to the form

$$\dot{\sigma}(t)\,\mathbf{S} = c_1(t)\sigma(t)\left(a^2\langle\mathbf{S},\mathbf{I}\rangle\dot{\boldsymbol{\varepsilon}}(t) + \frac{1}{\langle\mathbf{S},\mathbf{I}\rangle}\langle\mathbf{S},\dot{\boldsymbol{\varepsilon}}(t)\rangle\mathbf{S} + af(t)\|\dot{\boldsymbol{\varepsilon}}(t)\|\left(2\mathbf{S} - \frac{1}{3}\langle\mathbf{S},\mathbf{I}\rangle\mathbf{I}\right)\right) \quad (1)$$

  where $\langle\cdot,\cdot\rangle$ denotes the canonical scalar product in $\mathbb{R}^{3\times3}$, $\|\cdot\|$ Frobenius norm, the dot denotes the derivative with respect to t, $\mathbf{I}$ is the identity matrix and $a > 0$ is constant.
- By putting

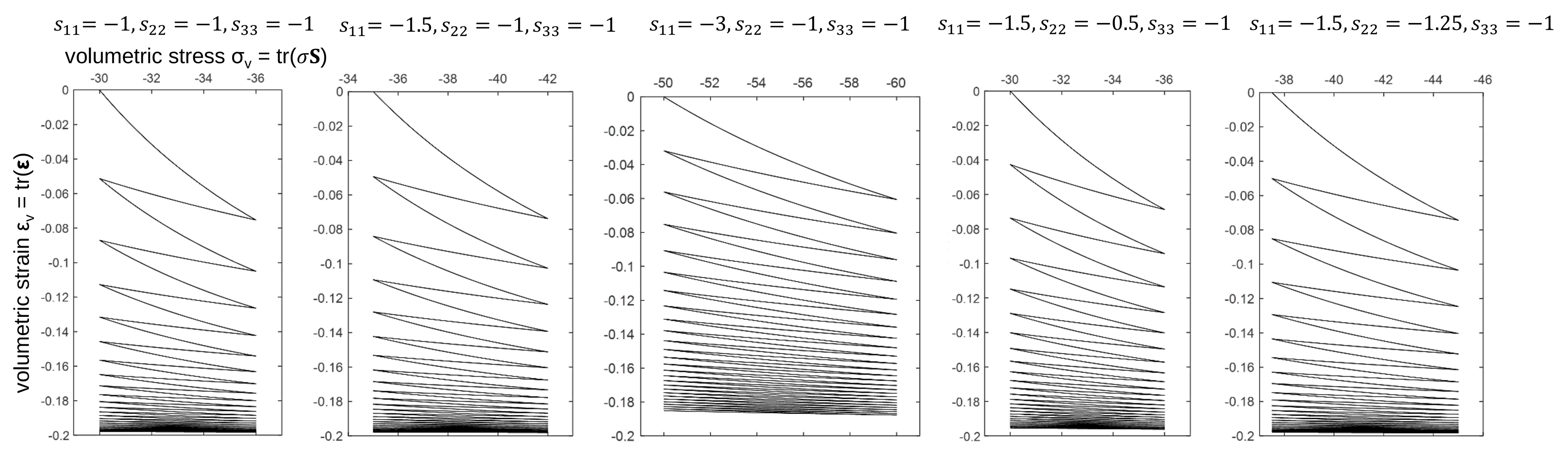
$$\mathbf{X}(t) = c_1(t)\frac{\sigma(t)}{\dot{\sigma}(t)}\dot{\boldsymbol{\varepsilon}}(t), \qquad \mathbf{Q} = \frac{\mathbf{S}}{\langle\mathbf{S},\mathbf{I}\rangle}, \qquad \mathbf{A} = \frac{\mathbf{Q}}{a^2 + \|\mathbf{Q}\|^2}, \qquad \mathbf{B} = \left(2 + \frac{1}{3a^2}\right)\mathbf{A} - \frac{1}{3a^2}\mathbf{I}$$

  and by further manipulation, we obtain

$$\mathbf{Q} = a^2\mathbf{X}(t) + \langle\mathbf{Q},\mathbf{X}(t)\rangle\,\mathbf{Q} - af(t)\|\mathbf{X}(t)\|\left(2\mathbf{Q} - \tfrac{1}{3}\mathbf{I}\right) \text{ for } \dot{\sigma} > 0,$$
$$\mathbf{Q} = a^2\mathbf{X}(t) + \langle\mathbf{Q},\mathbf{X}(t)\rangle\,\mathbf{Q} + af(t)\|\mathbf{X}(t)\|\left(2\mathbf{Q} - \tfrac{1}{3}\mathbf{I}\right) \text{ for } \dot{\sigma} < 0.$$

- $f = f(t) > 0$ is a given function of $e(t)$ in the form $f(t) = f_0\left(\frac{e(t)-e_d}{e_c-e_d}\right)^{\alpha}$, where $f_0 > 0$, $\alpha > 0$, $e_c > e_d > 0$ are assumed to be constant.
- $c_1(t) < 0$ is a parameter proportional to the elasticity modulus (in this case, material becomes rigid when $e(t)$ asymptotically converges to e_d), the parameter is assumed to depend on $e(t)$ in the form $c_1(t) = -\bar{c}(e(t) - e_d)^{-\beta}$, where $\bar{c} > 0$ and $\beta > 1$ are constants.
- Next equation in the computed system is the mass balance equation $\dot{e}(t) = (1 + e(t))\langle\dot{\boldsymbol{\varepsilon}}(t),\mathbf{I}\rangle$, where the void ratio $e(t) > e_d$.

gradual compaction of granular material under different anisotropic settings



ANISOTROPIC CASE OF LOADING

- First, the ODEs for the unknown function $e(t)$ in the form

$$\frac{\dot{e}(t)c_1(e(t))}{(1+e(t))\big(\langle\mathbf{A},\mathbf{I}\rangle + \phi(f(e(t)))\langle\mathbf{B},\mathbf{I}\rangle + \psi(f(e(t)))\langle\mathbf{B},\mathbf{I}\rangle\big)} = 1 \text{ for } \dot{\sigma} > 0,$$
$$\frac{\dot{e}(t)c_1(e(t))}{(1+e(t))\big(\langle\mathbf{A},\mathbf{I}\rangle + \phi(f(e(t)))\langle\mathbf{B},\mathbf{I}\rangle - \psi(f(e(t)))\langle\mathbf{B},\mathbf{I}\rangle\big)} = -1 \text{ for } \dot{\sigma} < 0,$$

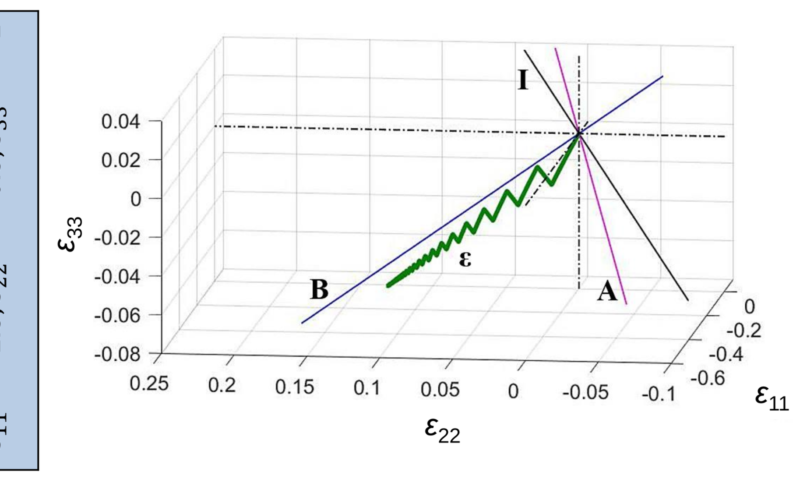
  are solved.
- Then the left-hand side of the ODEs for $\boldsymbol{\varepsilon}(t)$, see Model of Hypoplasticity, is known through

$$\mathbf{X}(t) = \mathbf{A} + \phi(f(t))\mathbf{B} + \psi(f(t))\mathbf{B} \text{ for } \dot{\sigma} > 0,$$
$$\mathbf{X}(t) = \mathbf{A} + \phi(f(t))\mathbf{B} - \psi(f(t))\mathbf{B} \text{ for } \dot{\sigma} < 0,$$

  where

$$\phi(f) = \frac{a^2f^2\langle\mathbf{A},\mathbf{B}\rangle}{1 - a^2f^2\|\mathbf{B}\|^2}, \psi(f) = af\,\frac{\sqrt{\|\mathbf{A}\|^2 - a^2f^2(\|\mathbf{A}\|^2\|\mathbf{B}\|^2 - \langle\mathbf{A},\mathbf{B}\rangle^2)}}{1 - a^2f^2\|\mathbf{B}\|^2}.$$

ratchetting (25 cycles) of $\varepsilon(t)$ with the anisotropic settings $s_{11} = -1.5, s_{22} = -0.5, s_{33} = -1$



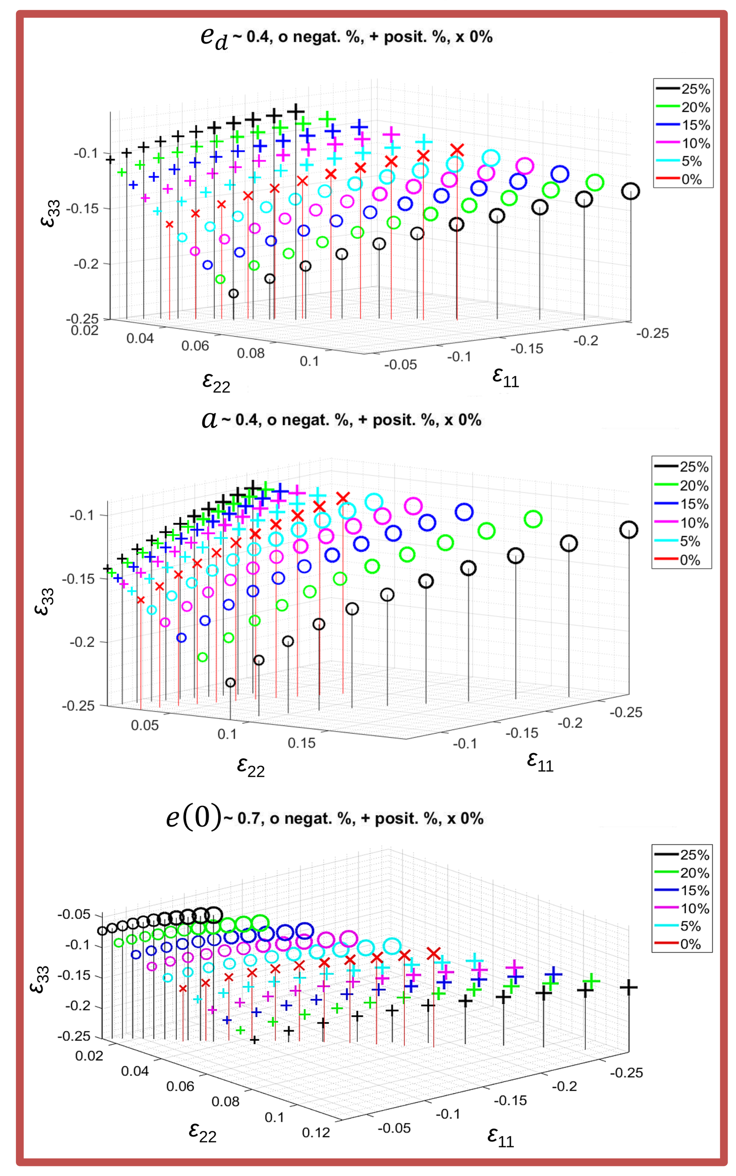
RESULTS OF UNCERTAINTY ANALYSIS

The results show that perturbations of nominal values of parameters from the first group (i.e. parameters e_c, α, $\bar{c}$, f_0, β) have only a weak impact on the final compaction. The effect of the uncertainty in the second group of parameters (i.e. a, e_d, $e(0)$) is much stronger.

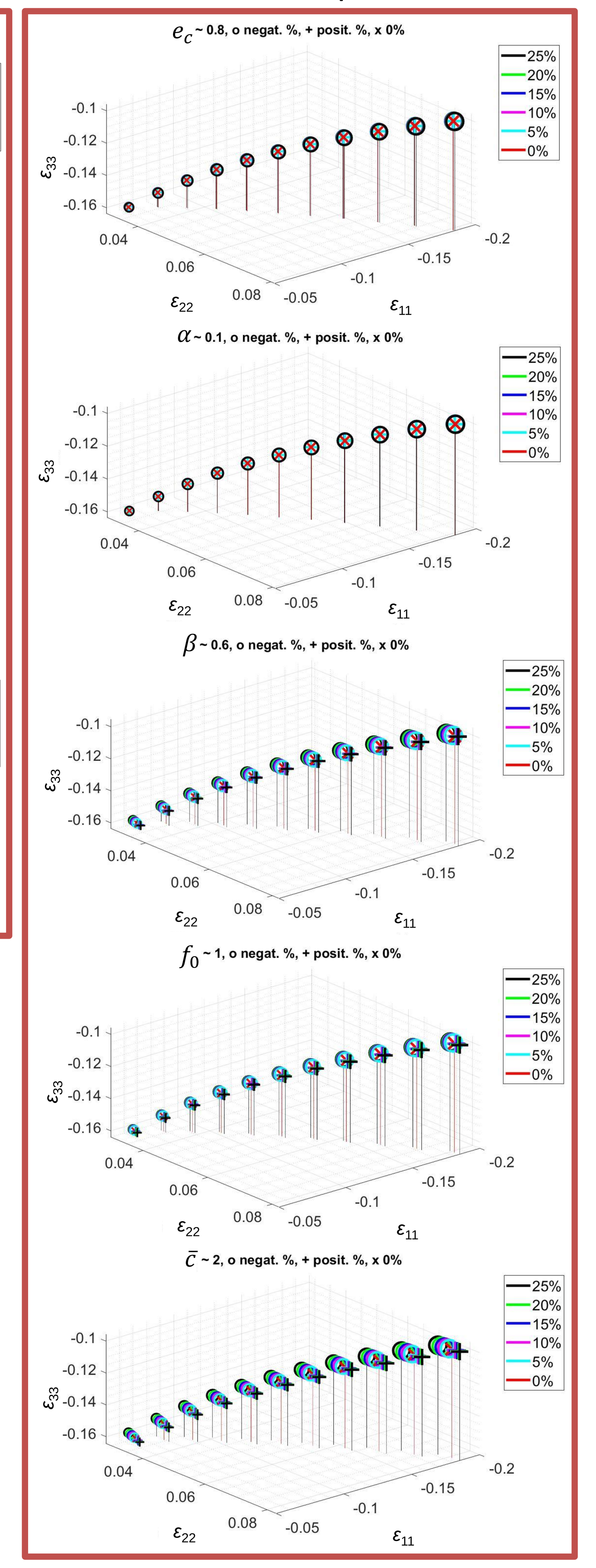
parameter	a	σ_1	σ_2	e_c	e_d	f_0	α	β	$\bar{c}$	$e(0)$	$\boldsymbol{\varepsilon}(0)$	n
nom. value	0.4	10	12	0.8	0.4	1	0.1	1.03	2	0.7	**0**	250

e(0), $\boldsymbol{\varepsilon}(0)$… initial conditions for $t = 0$; n… number of cycles; other parameters are mentioned in the text

Second Group

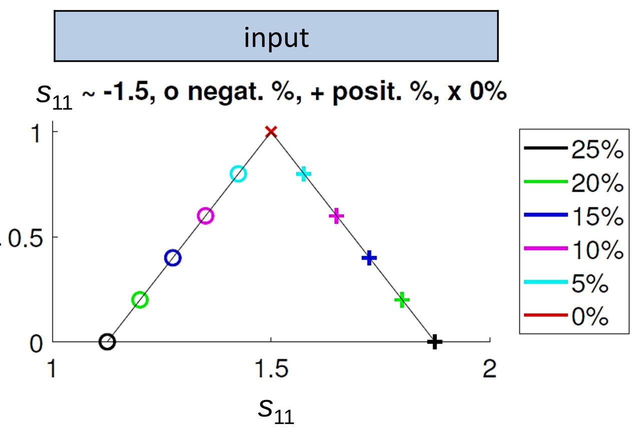


First Group

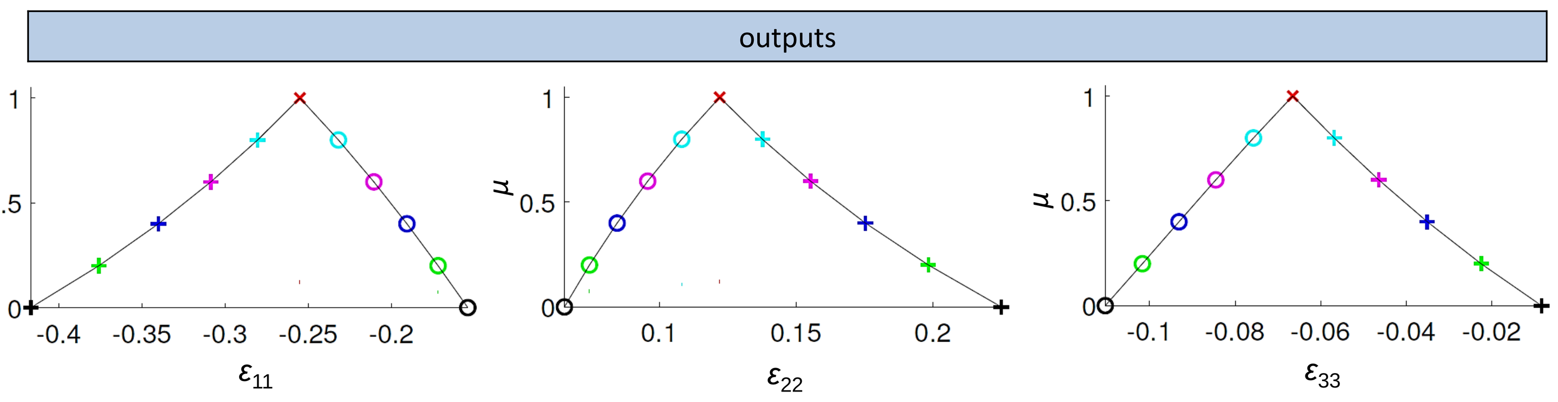


FUZZY SET INTERPRETATION

input



outputs



CONCLUSION

- The behaviour and the limit state of the strain ratchetting in the stress-controlled case of anisotropic loading and unloading cycles with different rates between the three principal stresses was calculated. The algorithm was implemented in Matlab.
- Weak changes of the model response to the parameters e_c, α, $\bar{c}$, f_0, β were detected in all the investigated settings of anisotropy.
- The uncertainty in the other parameters, i.e. a, e_d, $e(0)$, has a significant influence on the model response.
- The presented uncertainty analysis might help the experimenter to identify factors that deserve special attention.

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