Università della Svizzera italiana

Multilevel and Domain-Decomposition Solution Strategies for Solving Large-Scale Phase-Field Fracture Problems

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Phase-field fracture propagation for brittle fractures

$$(\boldsymbol{u}^t, c^t) = \underset{\boldsymbol{u}=\boldsymbol{u}^t_D \text{ on } \partial\Omega_D}{\arg\min} \ \Psi(\boldsymbol{u}, c) := \int_\Omega \left((1-c)^2 \ \psi_e^+(\boldsymbol{u}) + \psi_e^-(\boldsymbol{u}) \right) d\Omega + \frac{\mathcal{G}_c}{2} \int_\Omega \left(\frac{c^2}{l_s} + l_s |\nabla c|^2 \right) d\Omega + \frac{\gamma}{2} \int_\Omega (\langle c - c^{t-1} \rangle_-)^2 \ d\Omega$$

Discretization

- Unstructured grid
- Finite elements

Solution strategy

- Additive/Multiplicative Schwarz Preconditioned Inexact Newton method (ASPIN/MSPIN)
  → Domain-decomposition based approach
  → Nonlinear preconditioning
- Field Split preconditioning
  → Solve the displacement and phase field separately in preconditioning step

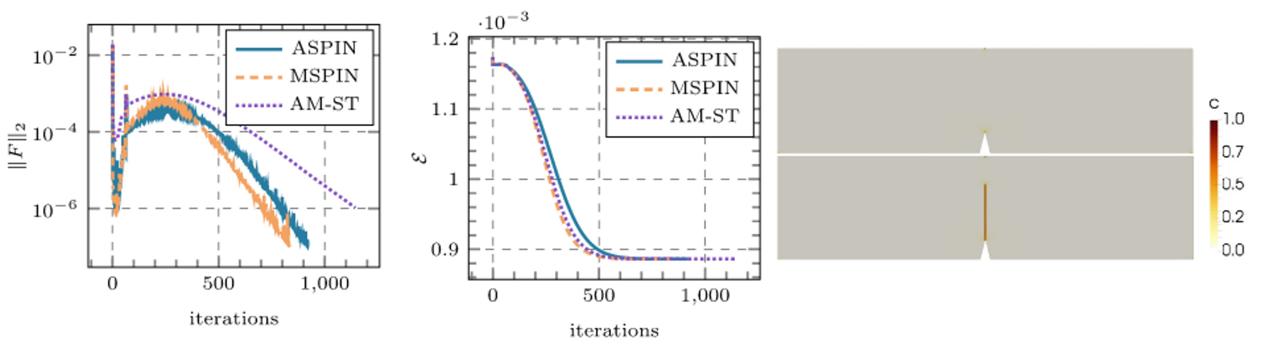
Idea of nonlinear preconditioning

- Original nonlinear problem:
  - Find (u, c) such that F(u, c) = 0
- Employ a preconditioning operator G
  - Such that solution of F(u, c) = 0 is same as G(F(u, c)) = 0
- The preconditioned nonlinear system can be written as a composite operator
  - $\mathscr{F}$(u, c) = G ∘ F(u, c) = 0
- Do a Newton iteration to solve $\mathscr{F}$(u, c) = 0.
  - This modifies the computation of preconditioned residual and preconditioned Jacobian

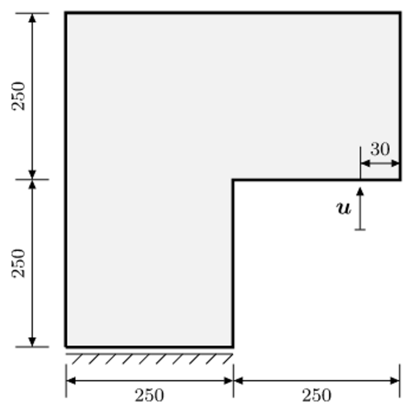
Numerical Tests

Compare different solution methods
- Alternate minimization (AM) method
  - Standard approach (AM-ST)
  - Newton Direct solver (AM-ND)
  - Newton Krylov (AM-NK)
- ASPIN (Additive preconditioning)
- MSPIN (Multiplicative preconditioning)

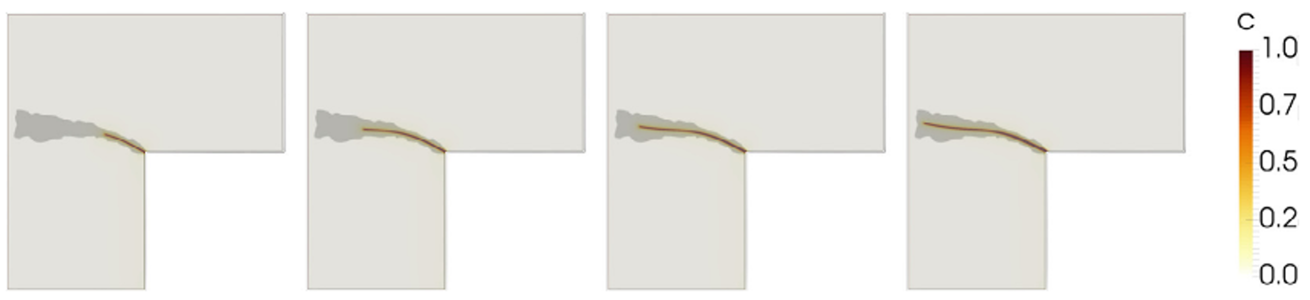
Convergence history of the iterative method



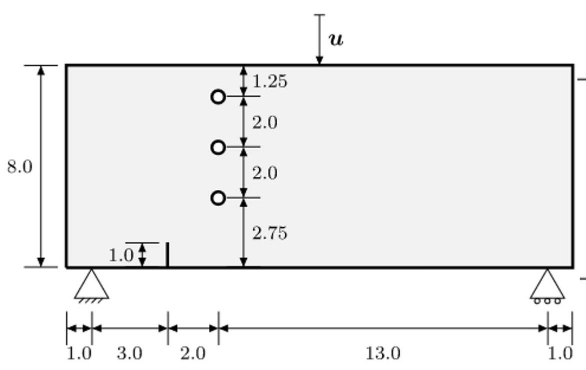
L-shaped panel test



Solver	Time (min)	Speedup with respect to				
		AM-ND	AM-NK	AM-INK	AM-ST	ASPIN
AM-ND	8,662.82	–	–	–	–	–
AM-NK	5,748.19	1.51	–	–	–	–
AM-INK	5,862.43	1.48	0.98	–	–	–
AM-ST	3,539.46	2.45	1.62	1.66	–	–
ASPIN	2,674.32	3.24	2.15	2.19	1.32	–
MSPIN	2,564.15	3.38	2.24	2.29	1.38	1.04

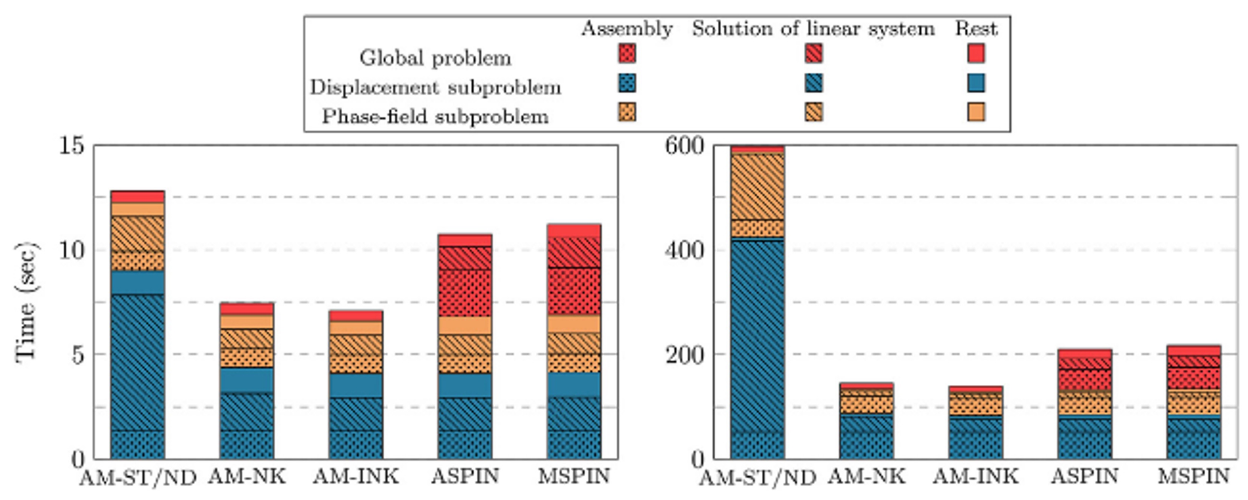


Asymmetrically notched beam test

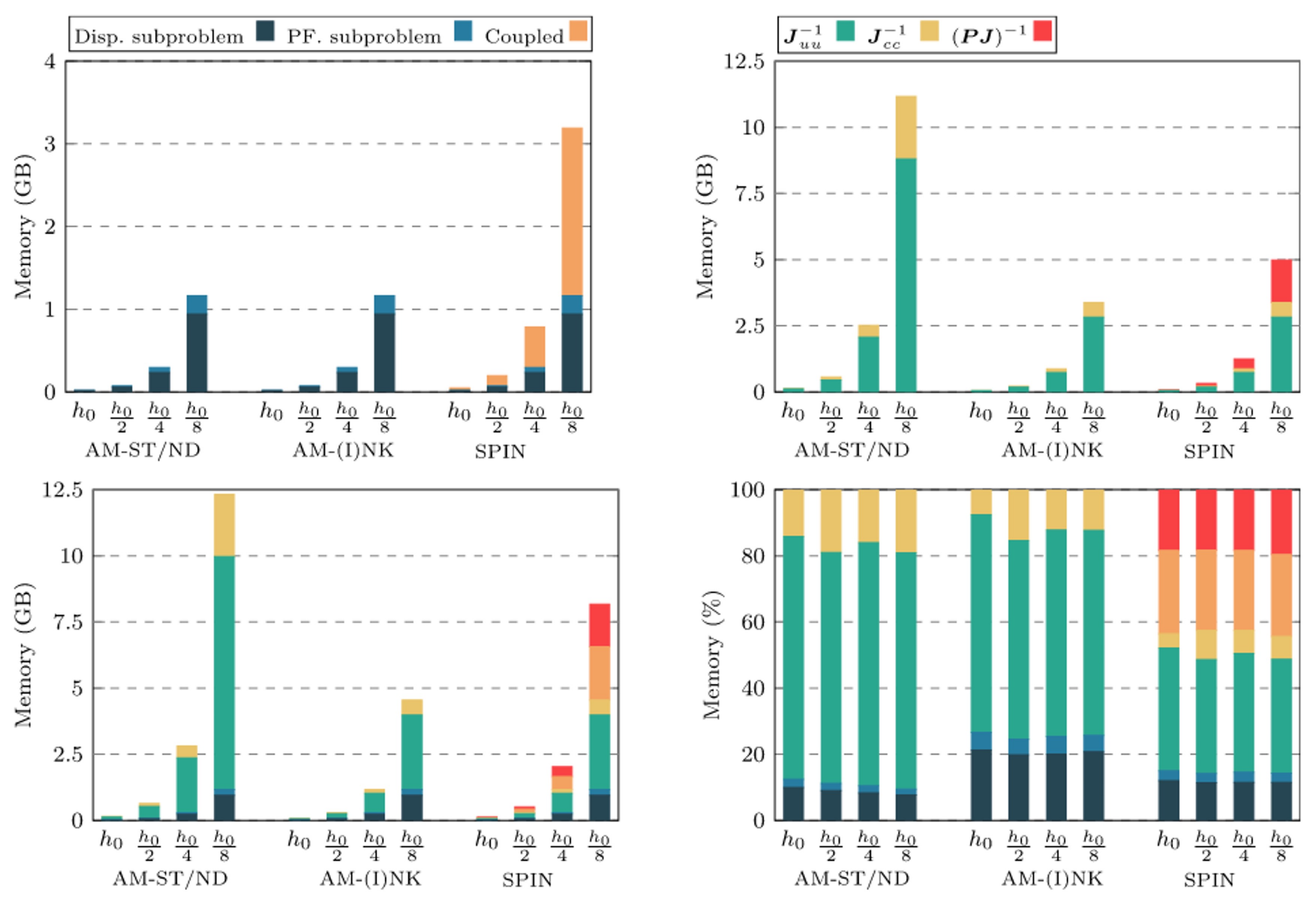


Solver	Time (min)	Speedup with respect to				
		AM-ND	AM-NK	AM-INK	AM-ST	ASPIN
AM-ND*	1,263.54	–	–	–	–	–
AM-NK*	1,060.75	1.19	–	–	–	–
AM-INK*	952.49	1.33	1.11	–	–	–
AM-ST*	563.77	2.24	1.88	1.69	–	–
ASPIN†	667.23	1.89	1.59	1.43	0.84	–
MSPIN+	710.64	1.78	1.49	1.34	0.79	0.94

Analysis of execution time



Analysis of memory requirements of iterative methods



Pressure induced phase-field fracture propagation

Challenging problem to solve

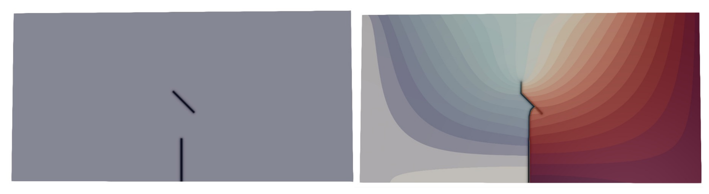
- Non-convex
- Non-smooth
- Ill-conditioned

$$E(\mathbf{u}, c, p) := \int_\Omega g(c) \ \psi_e(\boldsymbol{\varepsilon}(\mathbf{u})) + \frac{\mathcal{G}_c}{c_w} \left(\frac{w(c)}{l_s} + l_s |\nabla c|^2 \right) d\Omega - \int_{\Gamma_N} \bar{\mathbf{t}}_\Omega \cdot \mathbf{u} \ ds - \int_\Omega g(c) \nabla \cdot (p\mathbf{u}) \ d\Omega + \int_{\partial \Gamma_N} p\mathbf{n} \cdot \mathbf{u} \ ds,$$

$$c : \Omega \to [0, 1]$$

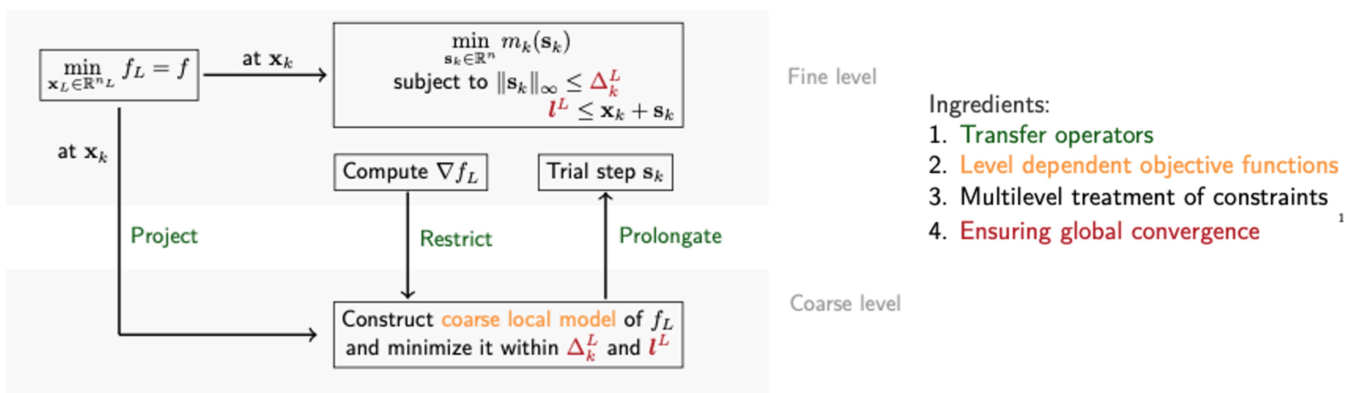
Discretization

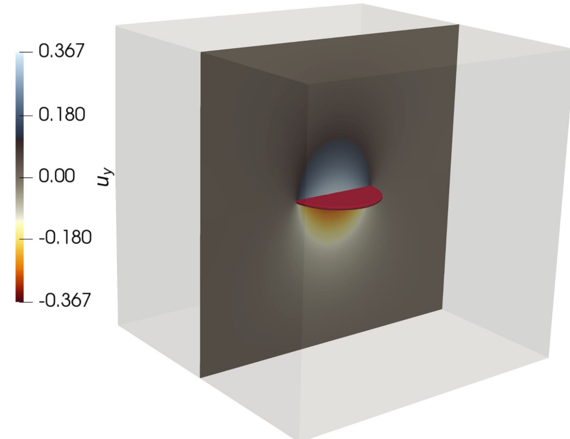
- Structured grid
- Finite elements



Solution strategy

- Recursive multilevel trust region (*RMTR*)
  → Globally convergent
- Sequential quadratic programming smoother (*MPRGP*, Projected Gauss-Seidel)





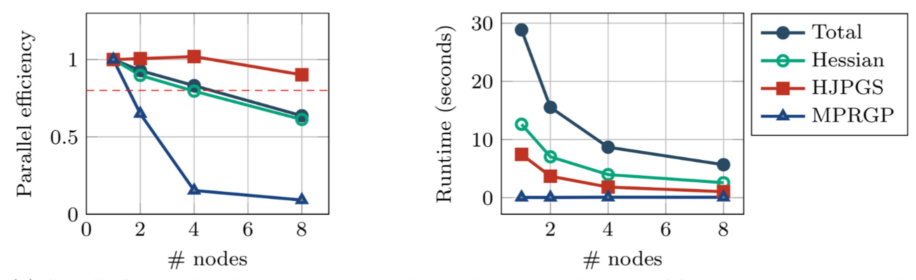
Sneddon test: Error of total crack volume (TCV) and opening displacement (COD)

# Dofs.	l_s	Err-TCV	Err-COD
202 612	1.08	70.08%	20.85%
1 826 132	0.52	17.80%	7.91%
15 479 572	0.25	5.48%	3.19%
127 420 052	0.13	0.34%	0.48%
434 125 332	0.08	0.055%	0.29%

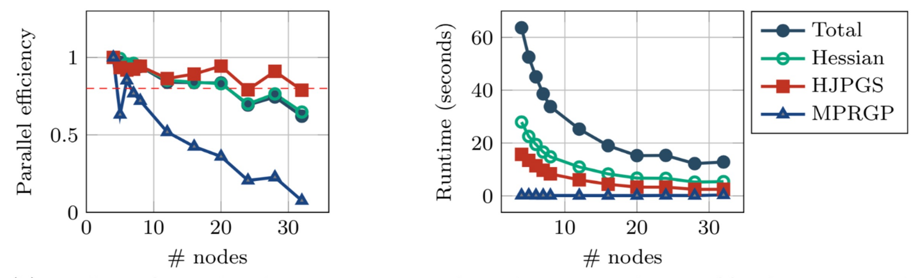
$$\mathrm{COD}(\mathbf{x}) := \frac{4pr(1-\nu^2)}{\pi E} \sqrt{1 - \left(\frac{\|\mathbf{x}\|_{\ell_2}}{r} \right)^2} \qquad \mathrm{TCV} := \frac{16\pi p r^3 (1-\nu^2)}{3E}$$

Strong scaling

- Dashed red line marks 80% efficiency



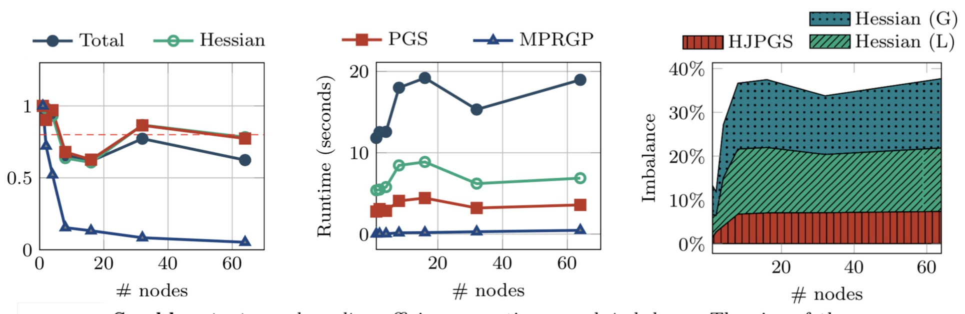
(a) **Small**: fine grid with $77 \times 77 \times 77$ nodes and 1 826 132 degrees of freedom, generated with 3 RMTR levels.



(b) **Medium**: fine grid with $157 \times 157 \times 157$ nodes and 15 479 572 degrees of freedom, generated with 3 RMTR levels.

Weak scaling

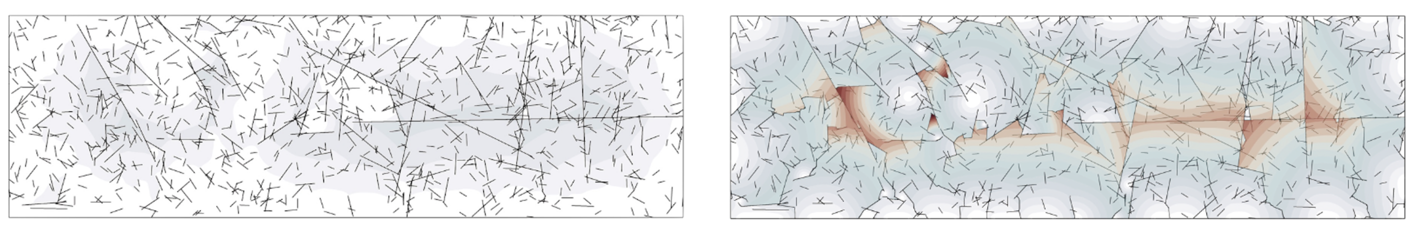
- Dashed red line marks 80% efficiency
- Imbalance due to hierarchy generated by refinement



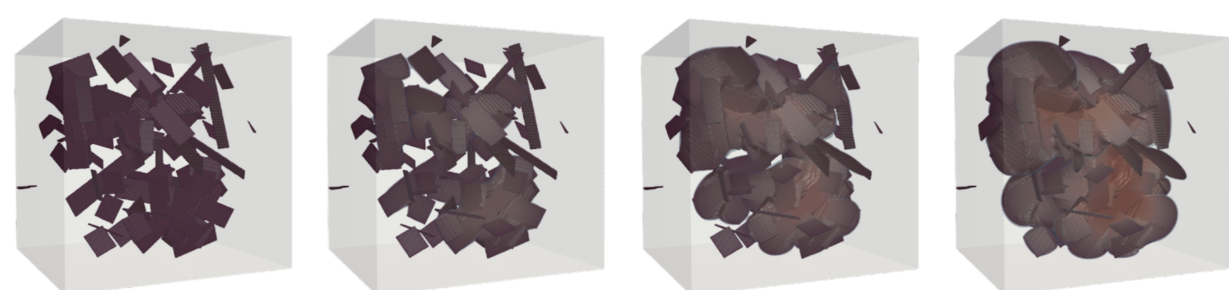
Sneddon test: *weak scaling* efficiency, runtimes, and *imbalance*. The size of the grid is $s \times s \times s$, where $s = \lceil (1000 \times n)^{\frac{1}{3}} \rceil$ hence with $s^3 \times 4$ dofs in the coarse level and $((s-1) \times 2^{(l-1)} + 1)^3$ at each RMTR level l (fine level is $l = 3$). The reminder rounded by the ceiling operator $\lceil \cdot \rceil$ will cause some fluctuations in the amount of work of each run and defects in the efficiency plot. The size of smallest run consists of 740 772 dofs while the largest one consists of 48 035 956 dofs.

Fracture networks in geological applications

2D: Reproducing the frequency and propagation of joints in sedimentary layers



2D: Two-dimensional simulation with **1000 randomly distributed fractures**



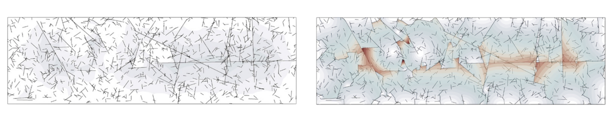
3D: Three-dimensional simulation with **100 randomly distributed fractures** and *242'793'828* dofs

Algorithmic scalability

SQP-Smoother

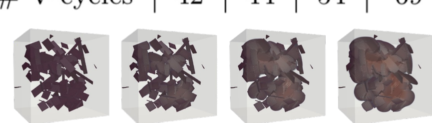
- *Hybrid Block Jacobi-Projected Gauss-Seidel*
- Each process performs *Projected Gauss-Seidel* on its local block
- Convergence properties are influenced by the number of blocks (36 cores/blocks per computing node)

# nodes	4	8	16	32
# V-cycles	126	135	147	154



2D: 1000 fractures, 28.1 M dofs

# nodes	25	50	75	100
# V-cycles	42	44	54	69



3D: 100 fractures, 122.7 M dofs

Open Access articles

Zulian, P., Kopaničáková, A., Nestola, M.G.C. *et al.* **Large scale simulation of pressure induced phase-field fracture propagation using Utopia**. *CCF Trans. HPC* (2021).

Kopaničáková, A., Kothari, H., Krause, R., 2023. **Nonlinear field-split preconditioners for solving monolithic phase-field models of brittle fracture**. Computer Methods in Applied Mechanics and Engineering, 403, p.115733.

Kopaničáková, A., Krause, R., 2020. **Recursive multilevel trust region method with application to fully monolithic phase-field models of brittle fracture**. Computer Methods in Applied Mechanics and Engineering, 360, p.112720.

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https://bitbucket.org/zulianp/utopia
https://bitbucket.org/alena_kopanicakova/pf_frac_spin

Platform for Advanced Scientific Computing

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