Multilevel and Domain-Decomposition Solution Strategies for Solving Large-Scale Phase-Field Fracture Problems

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Phase-field fracture propagation for brittle fractures

Discretization
- Unstructured grid
- Finite elements

Solution strategy
- Additive/Multiplicative Schwarz Preconditioned (ASPIN/MSPIN)
- Domain-decomposition based approach
- Nonlinear preconditioning
- Field Split preconditioning
- Solve the displacement and phase field separately in precondition step

Idea of nonlinear preconditioning
- Original nonlinear problem: Find $u^*$ such that $F(u^*, c^*) = 0$
- Employ a preconditioning operator $G$
- The preconditioned nonlinear system can be written as a composite operator $G(F(u, c) = 0)$
- Newton-Krylov (AM-ML)
- ASPIN (Additive preconditioning)
- MSPIN (Multiplicative preconditioning)

Original nonlinear problem:

- Find $(u, c)$ such that $F(u, c) = 0$
- This modifies the computation of preconditioned residual and preconditioned Jacobian

Convergence history of the iterative method

L-shaped panel test

Asymmetrical notch beam test

Analysis of execution time

Analysis of memory requirements of iterative methods

Convergence properties are influenced by the number of blocks (20 cores/blocks per computing node)

Presssure induced phase-field fracture propagation

Challenging problem to solve
- Nonconvex
- Non-smooth
- Inconditioned

Discretization
- Structured grid
- Finite elements

Solution strategy
- Recursive multilevel load region (RMTR)
- Globally convergent
- Sequential quadratic programming smoother (MRGSP, Projected Gauss-Seidel)

Numerical Tests

Comparing different solution methods
- Alternate minimization (AM-ML)
- Standard approach (AM-ST)
- Newton Direct solver (AM-ND)
- Newton-Krylov (AM-ML)
- ASPIN (Additive preconditioning)
- MSPIN (Multiplicative preconditioning)

Solutions strategy
- Recursive multilevel load region (RMTR)
- Globally convergent
- Sequential quadratic programming smoother (MRGSP, Projected Gauss-Seidel)

Strong scaling
- Dashed red line marks 85% efficiency

Weak scaling
- Dashed red line marks 85% efficiency
- Imbalance due to hierarchy generated for refinement

Fracture networks in geological applications

2D: Reproducing the frequency and propagation of joints in sedimentary layers

3D: Three-dimensional simulation with 100 fractures

3D: Three-dimensional simulation with 100 randomly distributed fractures and 242’793’828 dofs

Algorithmic scalability

- Hybrid Block Jacob-Projected Gauss-Seidel
- Each process performs Projected Gauss-Seidel on its local block
- Convergence properties are influenced by the number of blocks (20 cores/blocks per computing node)

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2D: 1000 fractures, 28.1 M dofs

3D: 300 fractures, 122.7 M dofs

https://bitbucket.org/zulianp/utopia