Institute of Computing

# Application of Deep and Reinforcement Learning to Boundary Control Problems

Zenin Easa Panthakkalakath, Juraj Kardoš, Olaf Schenk

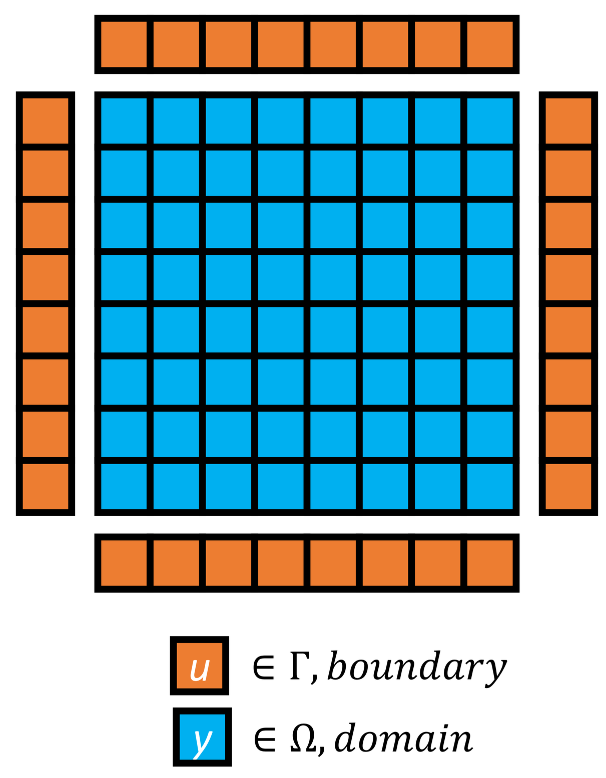
View on GitHub:

ABSTRACT

The goal of the boundary control problem is, in essence, to find the optimal values for the boundaries such that the values for the enclosed domain are as close as possible to desired values. Traditionally, the solution is obtained using nonlinear optimization methods, such as interior point, wherein the computational bottleneck is introduced by the large linear systems. Our objective is to use Deep Learning (DL) and Reinforcement Learning (RL) methods to solve boundary control problems with Dirichlet boundary condition faster than traditional solvers.

## What is an elliptic boundary control problem?

- Domain values follow an elliptic PDE.
- Domain cells have desired values with upper and lower bound constraints.
- Boundary cell values are controllable.




**Find the most optimal values for the boundaries such that the domain would be closest to their desired values, without violating their constraints.**

**In mathematical sense,**

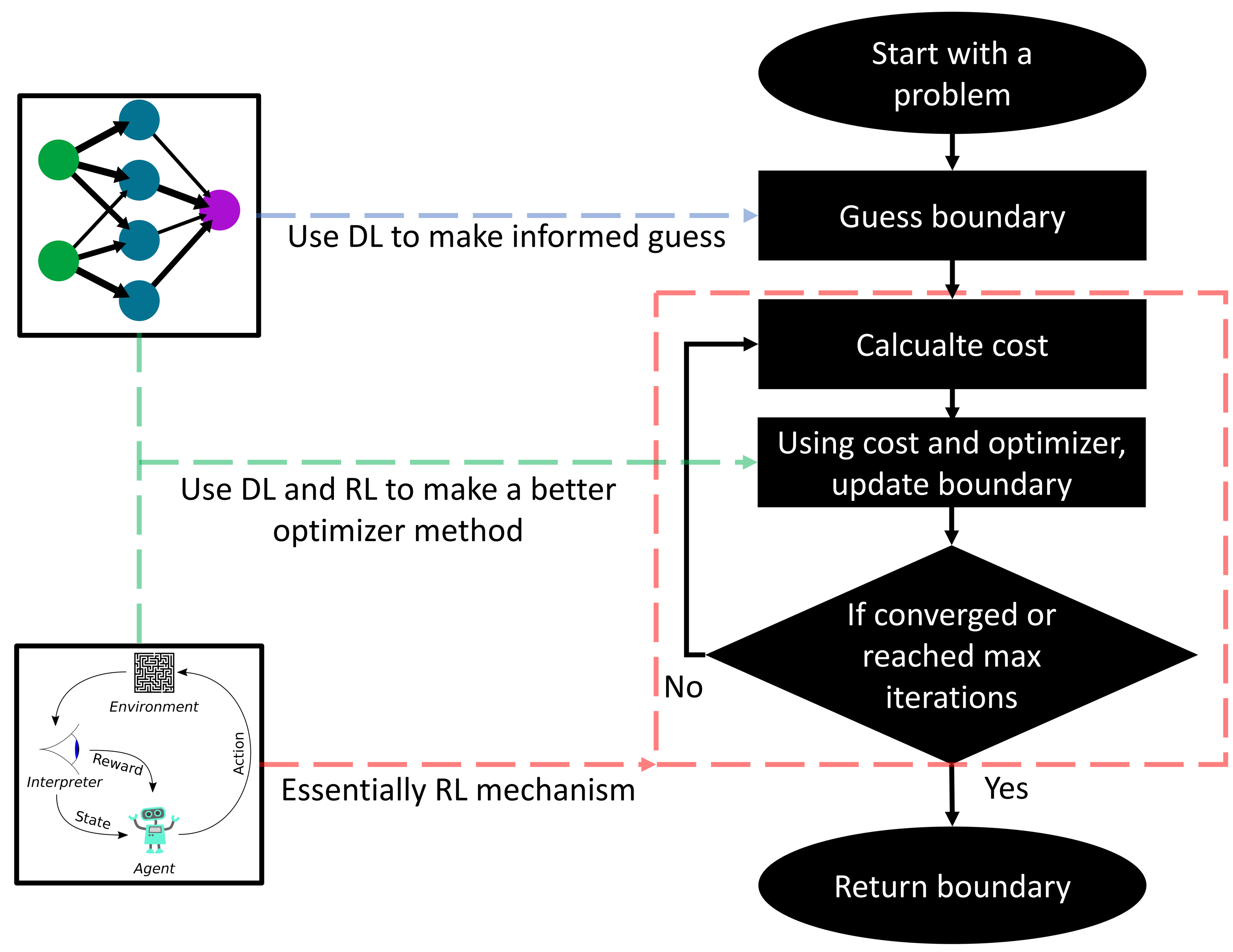
*Minimize:* $F(y,u) = \frac{1}{2}\int_{\Omega}\left(y(x) - y_d(x)\right)^2 dx + \frac{\alpha}{2}\int_{\Gamma}\left(u(x) - u_d(x)\right)^2 dx$

*Subject to:* $\nabla^2 y = c \;,\; y_{min} < y < y_{max} \;,\; u_{min} < u < u_{max}$

- $y$: domain values
- $y_d$: desired domain values
- $\alpha$: non-negative constant
- $u$: boundary values
- $u_d$: desired boundary values
- $c$: is a constant sourcing term

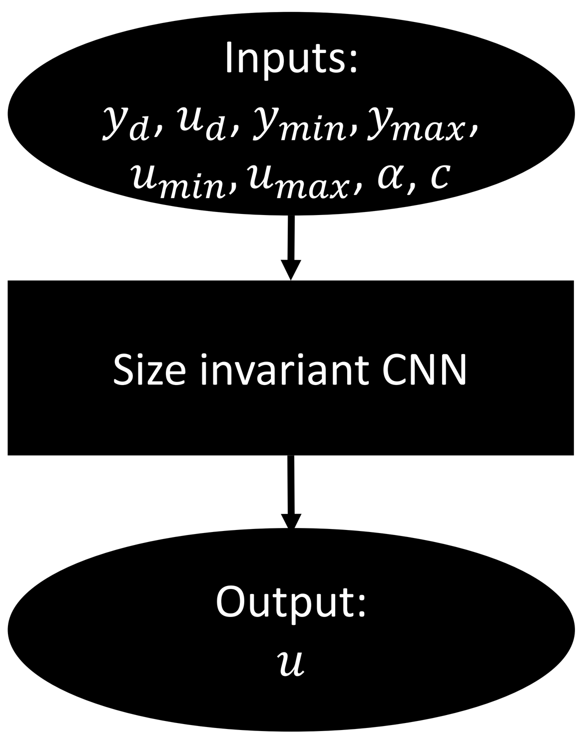
**Applications**: Heat transfer, fluid mechanics, acoustics, electromagnetism, structural mechanics, image processing.

## Where/how to use DL and RL?




**Note**: While designing a neural network, ensure that it can work with different input/output sizes, as the size of the domain/boundary can vary.

## Designing a method to make informed guess



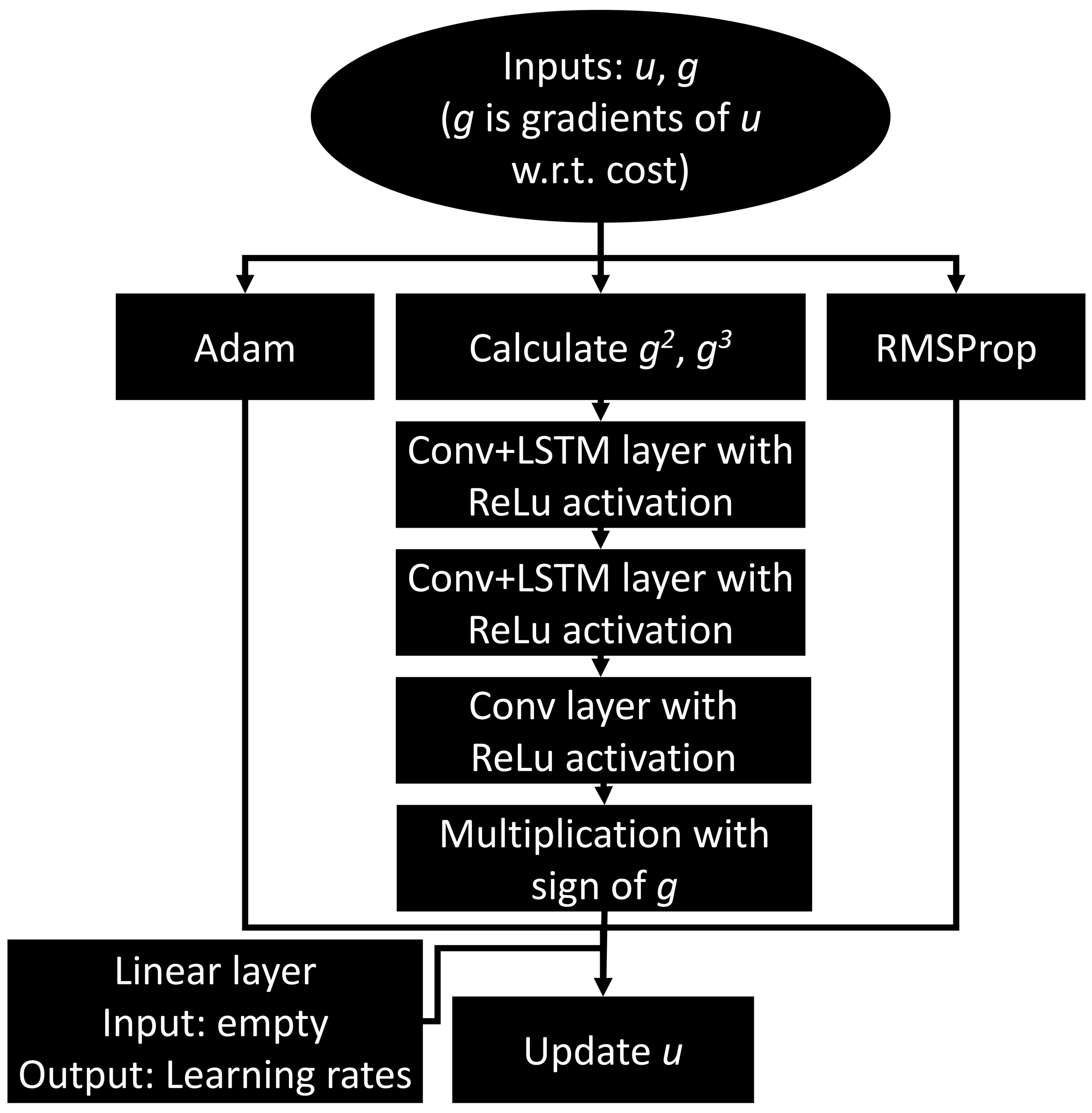

- A method for informed guess can only be considered to be good if the cost for the guessed boundary value is lower than the cost when the boundary values assume
  - the mean of the desired domain values.
  - the median of the desired domain values.
  - the values at the edges of the desired domain values.
- **We are still experimenting to find an initial guess network that can be considered good.**

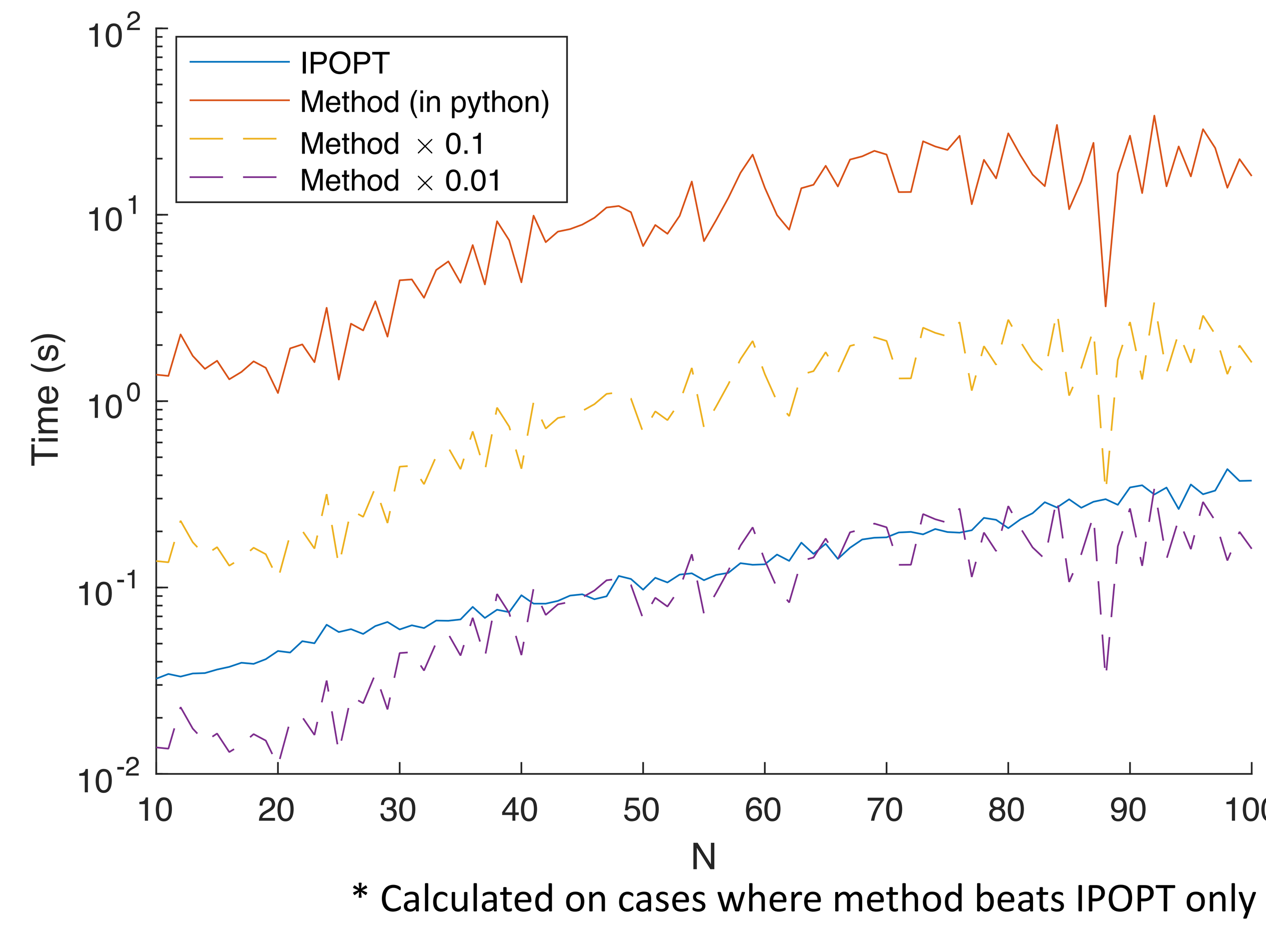
| | All mean | All median | Desired domain edge | Network | Total Wins |
|---|---|---|---|---|---|
| Mean | - | 7975 | 5857 | 5094 | **18926** |
| Median | 2190 | - | 5790 | 5019 | **12999** |
| Desired domain edge | 4143 | 4210 | - | 1959 | **10312** |
| Network | 4906 | 4981 | 8105 | - | **17992** |

**Note:** The best network is still not as good as using mean values as starting point.

## Designing a method to iteratively perform optimization

- Both spatial and temporal variations need to be considered.
- The input would be boundary values and it's corresponding gradients calculated using the cost function.
- CNNs, RNNs and transformers could be used.
- **The flowchart shows a method that performs reasonably well.**




* Calculated on cases where method beats IPOPT only

| | # cases in which feasible solution is found | # cases one has lower cost than the other |
|---|---|---|
| **IPOPT** | 5907 | 3962 |
| **Method** | 5108 | 1945 |

* Tested on 10,000 cases

REFs.

- Wächter, Andreas, and Lorenz T. Biegler. "On the implementation of an interior-point filter line-search algorithm for large-scale nonlinear programming." Mathematical programming 106 (2006): 25-57.
- Bello, Irwan, et al. "Neural optimizer search with reinforcement learning." International Conference on Machine Learning. PMLR, 2017.

https://www.ci.inf.usi.ch/

zenin.easa.panthakkalakath@usi.ch, juraj.kardos@usi.ch , olaf.schenk@usi.ch