Geodynamo simulations in a full sphere

Fabian Burmann[1], Jiawen Luo[1], Philippe Marti[1], Andrew Jackson[1],
1 Institute of Geophysics, ETH Zurich;

1 Introduction

Although the geomagnetic field exists since about 4 Gyr, recent estimates for the formation of the Earth's inner core go back no further than 500 Myr to 1 Gyr. Here we run rapidly rotating dynamos in a full sphere geometry, representative of the Earth's dynamo before the nucleation of the inner core. Numerically, the full sphere bears the difficulty of an adequate treatment of the singularity at the center. We perform a set of numerical simulations using a fully spectral simulation framework, where a careful choice of the radial basis functions resolves this singularity.

2 Method Overview

Governing non dimensional equations for the velocity field $\mathbf{u}$ magnetic field $\mathbf{B}$ and temperature field Θ in the full sphere:

$$(\partial_t - \mathrm{Pm}\nabla^2)\mathbf{u} = \mathbf{u}\times(\nabla\times\mathbf{u}) + \frac{\mathrm{Pm}^2\mathrm{Ra}}{\mathrm{EPr}}\Theta\mathbf{r} - \frac{\mathrm{Pm}}{\mathrm{E}}\mathbf{z}\times\mathbf{u} - \frac{\mathrm{Pm}}{\mathrm{E}}(\nabla\times\mathbf{B})\times\mathbf{B} - \nabla\mathbf{p}$$

$$(\partial_t - \nabla^2)\mathbf{B} = \nabla\times(\mathbf{u}\times\mathbf{B})$$

$$\left(\partial_t - \frac{\mathrm{Pm}}{\mathrm{Pr}}\nabla^2\right)\Theta = \mathbf{u}\cdot\mathbf{r} - \mathbf{u}\cdot\nabla\Theta$$

- With the non-dimensional control parameters:

$$\mathrm{Pr} = \frac{\nu}{\kappa},\quad \mathrm{Pm} = \frac{\nu}{\eta},\quad \mathrm{Ra} = \frac{g_0\alpha\beta r_0^2}{2\Omega\kappa},\quad \mathrm{E} = \frac{\nu}{2\Omega r_0^2}.$$

- Boundary Conditions: no -slip boundary condition for $\mathbf{u}$, insulating boundary condition for $\mathbf{B}$ and a fixed flux boundary condition for Θ.

The numerical simulations are conducted using the fully spectra simulation framework **QUICC** [1,2]:

- Toroidal-poloidal decomposition:

$$\mathbf{u} = \nabla\times\nabla\times\mathcal{S}_u(r,\theta,\phi)\mathbf{r} + \nabla\times\mathcal{T}_\mathbf{u}(\mathbf{r},\theta,\phi)\mathbf{r}$$
$$\mathbf{B} = \nabla\times\nabla\times\mathcal{S}_B(r,\theta,\phi)\mathbf{r} + \nabla\times\mathcal{T}_\mathbf{B}(\mathbf{r},\theta,\phi)\mathbf{r}$$

- Above scalars, including the temperature field Θ are further expanded by spherical harmonics in horizontal direction and Jones-Worland polynomials are obtained by combining the r^l factor and a one-sided Jacobi polynomial with even argument

$$W_n^l(r) \propto r^l P_n^{(\alpha,\beta)}(2r^2-1)$$

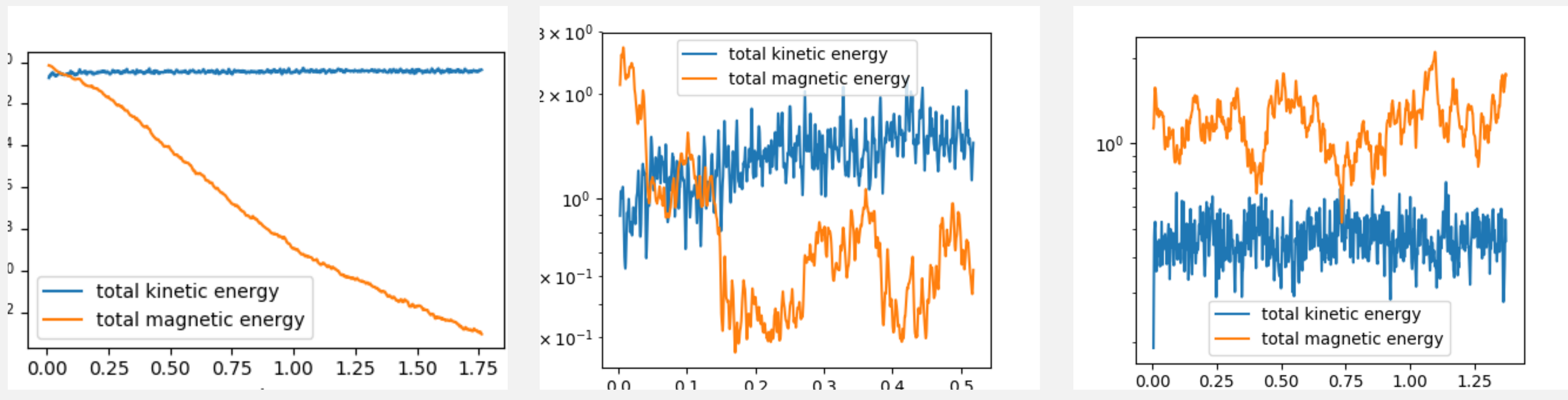
- Here, $W_n^l(r)$ guarantees infinite differentiability of all the scalars and resolves the artificial singularity at the center of the sphere in spherical coordinates. We believe this critical behaviour is the key for success of the numerical scheme in a full sphere, We fix the choice of $\alpha = -\frac{1}{2}$ and $\beta = l - \frac{1}{2}$

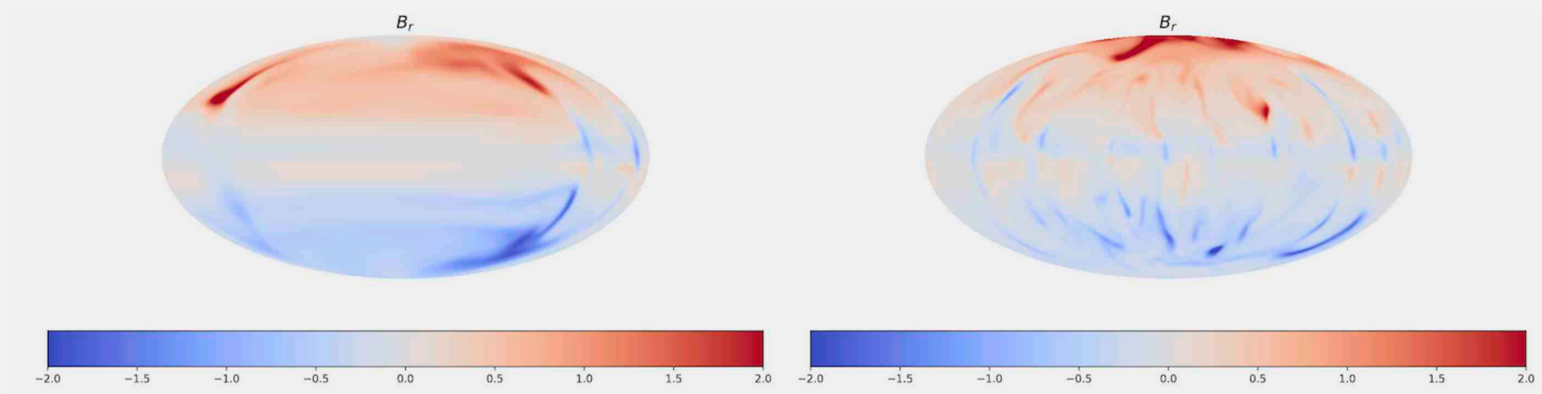
The numerical code is massively parallel and efficiently scalable up to several 1000 cores, which allows us to calculate a set of numerical dynamos that are representative of the Earth's palaeomagnetic field prior to the formation of the inner core.

3 Results

Depending on E, Ra and Pm, we observe three different regimes for the dynamo:

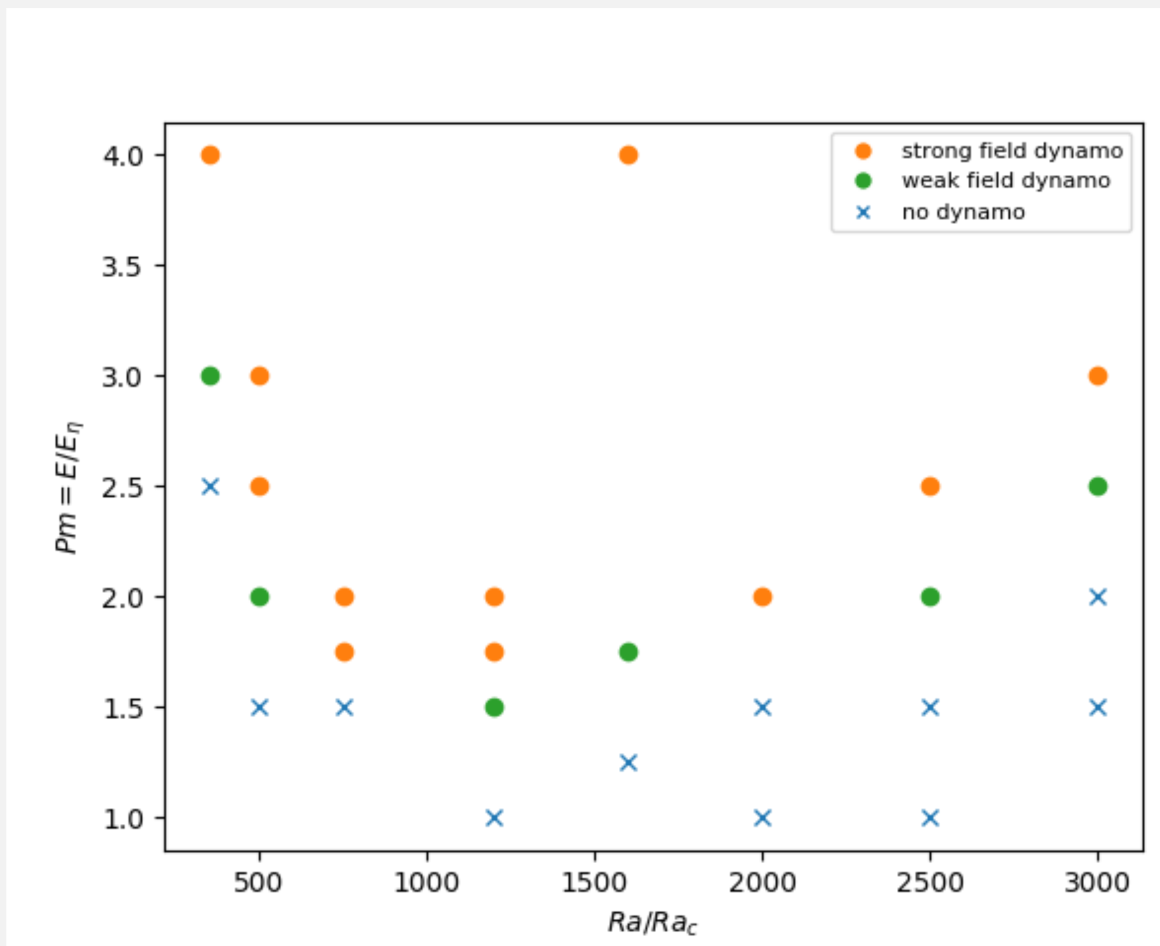
Failed dynamos | Weak field dynamos | Strong field dynamos





Snapshots of the magnetic field for two dynamos

Observed dynamo regimes as a function of super criticality Ra/Ra_c and magnetic Prandtl number Pm for Ekman number $E = 10^{-4}$.



4 Discussion and Conclusion

- Utilising the fully spectral simulation framework QuICC allows us to run the first series of dynamo simulations in a full sphere geometry.
- Results show that the dynamo regime for full sphere dynamos is more complex compared to spherical shells [3].
- More simulations at larger Ra and smaller E will be run in the furore which will complete the picture of the dynamo remise and allow a scaling analysis.

[1] P.Marti, M.A.Calkins and K.Julien. "A computationally efficient spectral method for modelling core dynamics", Geochemistry, Geophysics, Geosystems, 17(8):3031-3053,2016.

[2] P.Marti, A. Jackson. "Accurate and efficient Jones_worland spectral transforms for planetary applications". Proceedings of the Platform for Advanced Scientific Computing Conference, pages 1-10, 2021

[3] U.R. Christensen and J.Aubert "Scaling properties of convection-driven dynamos in rotating spherical shells and application to planetary magnetic fields",geophysical Journal International, 166(1):97-114, 2006.